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# NUMERICAL STUDY OF THE INTERACTION OF A SUPERSONIC GASEOUS JET WITH A PLANAR TARGET

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*Various regimes of interaction of an axisymmetric supersonic underexpanded jet of a gas with both a finite and an infinite planar target are studied numerically. The model of an ideal perfect gas and one variant of Godunov's highly accurate scheme are used. The calculated and experimental frequency spectra of pressure oscillations in the center of the target are compared: their good agreement is observed.*

## INTRODUCTION

The interaction of an axisymmetric supersonic jet with a planar target perpendicular to the jet axis has been studied rather extensively both experimentally and theoretically (see, for instance [1–6]). Nevertheless, there is some disagreement between the results of numerical calculations and physical experiments, especially for an infinite planar target. The reason for this disagreement is the fact that most researchers used a finite-size target (of the order of the nozzle diameter) in their numerical experiments. In this case, a regime of self-induced oscillations with a small number of harmonics and a clear fundamental tone with a significant amplitude is observed, which makes it possible to compare the results obtained with a physical experiment. For an infinite (of significant length) target, it follows from the results of the physical experiment that the autooscillatory mechanism is expressed less clearly and is characterized by a complex frequency spectrum, which it is difficult to be reproduce in a numerical model (difference scheme). Note that the results of independent physical experiments conducted using different equipment are in good agreement. This confirms the credibility of these results. The reason for the discrepancy between the numerical results and experimental data may be the quality of the numerical method used and the formulation of the boundary conditions.

In the present work, the numerical model is tested on two autooscillatory processes (with known characteristics) of interaction of a jet and a target located normal to it. The calculation results are compared with the data of independent physical experiments. The choice of geometric dimensions of the computational domain and the target corresponds to the formulation of the problem in one physical experiment or another.

## 1. NUMERICAL MODEL

**1.1. Formulation of the Problem (Boundary and Initial Conditions).** A conical nozzle of radius OA $\equiv r_A = 1$ (after the corresponding normalization) with a semi-apex angle $\theta$A is located at a distance $h$ from a planar target DF, which is located normal to the nozzle (Fig. 1). The supersonic exhaustion from this nozzle (segment OA of the computational domain boundary) is modeled by a flow from an isentropic

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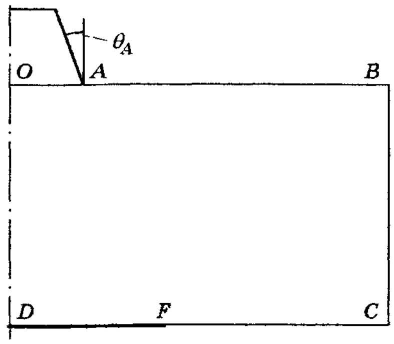


Fig. 1

source. The segment OD of the computational domain boundary is the axis of symmetry. The segments of the "boundary" AB, BC, and CF are "transparent" for disturbances leaving the domain. Thompson's "nonreflecting" conditions were imposed on them [7]. The change in the position of the point F separating different segments of the boundary DC simulates the finite size of the target (the limiting case DF = DC corresponds to an infinite target).

Initially, the space around the nozzle is filled by a stationary gas whose state is determined by the pressure $p_\infty$, density $\rho_\infty$, and ratio of specific heats $\gamma$. Using a given Mach number at the nozzle exit MA, jet pressure ratio $n$, and reduced parameters of the ambient space, we calculate a steady flow from the above-mentioned source. In fact, this procedure reduces to determination of the Mach number M in an arbitrary point of the segment OA of the computational domain boundary (Fig. 1).

**1.2. Governing Equations.** To describe unsteady spatial (with planar and axial symmetry) flows of an inviscid heat-non-conducting gas, we write the system of gas-dynamic equations in a divergent form:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial x} + \frac{\partial \boldsymbol{G}(\boldsymbol{U})}{\partial y} = -\boldsymbol{H},$$

$$\boldsymbol{U} = (\rho, \rho u, \rho v, e)^{\mathrm{t}}, \quad \boldsymbol{F}(\boldsymbol{U}) = (\rho u, \rho u^2 + p, \rho uv, (e+p)u)^{\mathrm{t}},$$

$$\boldsymbol{G}(\boldsymbol{U}) = (\rho v, \rho uv, \rho v^2 + p, (e+p)v)^{\mathrm{t}}, \quad \boldsymbol{H}(\boldsymbol{U}) = \frac{\nu}{y}(\rho v, \rho uv, \rho v^2, (e+p)v)^{\mathrm{t}}, \tag{1}$$

$$e = \rho\Big(\varepsilon + \frac{u^2+v^2}{2}\Big), \qquad \varepsilon(p,\rho) = \frac{p}{\rho(\gamma-1)}.$$

Here $\rho$ is the density, $u$ and $v$ are the velocity components along the $x$ and $y$ axes, $p$ is the pressure, $e$ is the total energy, and $\varepsilon$ is the specific internal energy; $\nu = 0$ and 1 refer to the planar and axial symmetry, respectively.

**1.3. Method of Solution.** It is known [1, 3] that the method of identification of discontinuities is perfect for numerical description of inviscid heat-non-conducting gas flows from the viewpoint of accuracy (approximation errors). As is shown in [3], this method can be used almost without any limitations for two-dimensional steady supersonic flows with capturing (and possible filtration) of all gas-dynamic discontinuities, including the weak ones. In our case, however, the use of this approach, as with the approach proposed by Godunov et al. [1] (with capturing of the "main" discontinuities only) is not justified because of the dramatic increase in the complexity of the algorithm of identification of a prior unknown structure of discontinuities. The shock-capturing method described below was used to solve this problem.

For numerical integration of system (1), we used a quasi-monotonic difference scheme of high accuracy [8], which is a modification of the known two-dimensional difference scheme proposed by S. K. Godunov [1]. We briefly describe the scheme used.

The difference analog of system (1) on a rectangular grid may be written in the form of the following two-step difference scheme:

$$U_{i,j}^{n+1} = U_{i,j}^{n} - \frac{\tau}{h_x}(F_{i+1/2,j}^{n} - F_{i-1/2,j}^{n}) - \frac{\tau}{h_y}(G_{i,j+1/2}^{n} - G_{i,j-1/2}^{n}) - \frac{\nu\tau}{y_{i,j}} H_{i,j}^{n}. \qquad (2)$$

Here $h_x \equiv \Delta x = x_{i+1/2,j} - x_{i-1/2,j}$, $h_y \equiv \Delta y = y_{i,j+1/2} - y_{i,j-1/2}$, and $\tau = t^{n+1} - t^n$ is the step of the grid. The subscripts $i$ and $j$ correspond to the values of the quantities at the cell centers ("small" quantities in Godunov's scheme), $i \pm 1/2$ and $j \pm 1/2$ refer to the values at the ribs of the grid ("large" quantities in Godunov's scheme), and $n$ corresponds to the values of the quantities in the time layer $t = n\tau$. To increase the order of accuracy in spatial variables in our variant of the scheme, we used a piecewise-continuous approximation of the solution with a linear distribution of the quantities over the cell instead of a piecewise-constant approximation (Godunov's scheme). "Large" quantities, for example, $(R, U, P)_{i+1/2,j}$ and $(R, V, P)_{i,j+1/2}$, which enter the components of the auxiliary vectors $F = F(m_L, m_R)$ and $G = G(g_L, g_R)$, are calculated by solving the corresponding one-dimensional problem of the decay of an arbitrary discontinuity for the following input parameters:

$$m_{L,i+1/2,j} = (\rho, u, p)^{\mathrm{t}}_{L,i+1/2,j}, \qquad m_{R,i+1/2,j} = (\rho, u, p)^{\mathrm{t}}_{R,i+1/2,j},$$

$$g_{L,i,j+1/2} = (\rho, v, p)^{\mathrm{t}}_{L,i,j+1/2}, \qquad g_{R,i,j+1/2} = (\rho, v, p)^{\mathrm{t}}_{R,i,j+1/2}.$$

In accordance with the accepted linear distribution of the quantities over the cell, the expressions for the vector components are written as

$$m_L = m_{i,j} + \frac{1}{2}\Phi(r_{i,j})(m_{i,j} - m_{i-1,j}), \qquad m_R = m_{i+1,j} - \frac{1}{2}\Phi(r_{i+1,j}^{-1})(m_{i+1,j} - m_{i,j}),$$

$$g_L = g_{i,j} + \frac{1}{2}\Phi(s_{i,j})(g_{i,j} - g_{i,j-1}), \qquad g_R = g_{i,j+1} - \frac{1}{2}\Phi(s_{i,j+1}^{-1})(g_{i,j+1} - g_{i,j}),$$

where $r_{i,j} = (m_{i+1,j} - m_{i,j})/(m_{i,j} - m_{i-1,j})$ and $s_{i,j} = (g_{i,j+1} - g_{i,j})/(g_{i,j} - g_{i,j-1})$; the subscripts $L$ and $R$ correspond to the left and right (upper and lower) vicinities of a given rib.

The function $\Phi(r)$ plays the role of a limiter of the derivative and a monotonizator of the solution. As is shown by Spekreijse [9], depending on the form of the function $\Phi(r)$, we can construct a family of "monotonic" difference schemes, which have the second order in spatial variables in the region of the smooth solution and a high degree of localization of gas-dynamic discontinuities. In our calculations, we used the limiter $\Phi(r) = (r + |r|)/(1 + |r|)$ as the function $\Phi(r)$.

The step of integration in time $\tau$ is chosen in the course of computation at each time layer from the conditions of stability for the two-dimensional scheme [1]: $\tau = K\tau_x\tau_y/(\tau_x + \tau_y)$, where $0 < K < 1$, and $\tau_x$ and $\tau_y$ are time intervals during which the waves formed in the Riemann problem reach the cell boundary along the $x$ and $y$ axes, respectively.

## 2. NUMERICAL SIMULATION

**2.1. Autooscillatory Flow upon Interaction of a Jet with a Finite Target.** The calculation was performed for a jet with the following parameters: Mach number at the nozzle exit $M$A $= 2$, jet pressure ratio $n = 3$, ratio of specific heats $\gamma = 1.4$, nozzle radius $r$A $= 15$ mm, and semi-apex angle of the nozzle $\theta$A $= 10°$. A planar target of size $r_F = 6.5r$A was located in a steady flow at a distance of $h = 7.3r$A, $7.5r$A, and $8.0r$A from the nozzle exit. The calculation was performed on a uniform grid $\Delta x = \Delta y = 0.05r$A. The calculation results for $h = 8r$A are plotted in Figs. 2 and 3 [the graph of oscillations of dimensionless pressure on the target at the stagnation point is shown in Fig. 2 and the frequency spectrum of the process $p(t)$ is shown in Fig. 3].

One result of the conducted series of calculations (for different $h$) is the satisfactory agreement between the fundamental frequency of the simulated process and the corresponding frequency in the experiments [5, 6] (see Table 1). Apart from the fundamental frequency, additional frequencies were revealed, which were not observed in [5, 6], possibly because of the time lag of the measurement equipment used and the roughness of the numerical method (method of coarse particles).

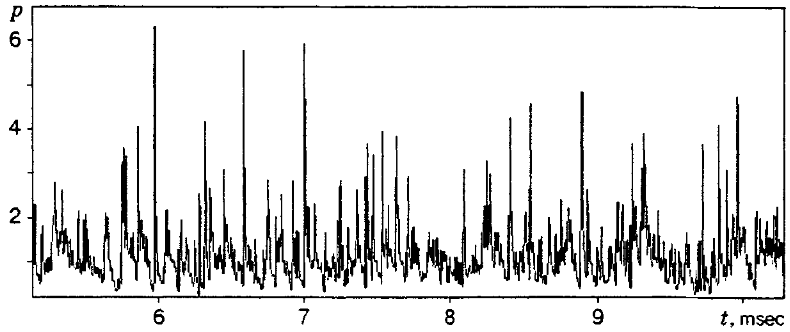

Fig. 2

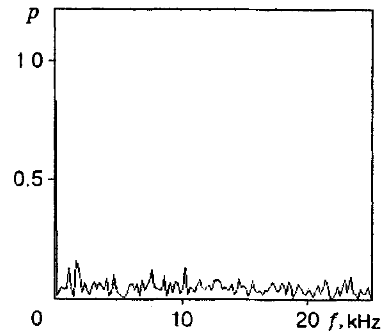

Fig. 3

TABLE 1

| $h$ | Experimental frequencies, kHz | | Calculated frequencies, kHz |
|---|---|---|---|
| | [5, 6] | [10] | |
| 7.3$r$A | 1.33 | — | 1.556, 2.140, 7.976, 11.868 |
| 7.5$r$A | 1.33 | — | 1.315, 1.972, 4.932, 11.837 |
| 8.0$r$A | 1.18 | — | 1.167, 1.751, 7.587, 10.117 |
| 6.39$r$A | — | 10.01, 19.531, 29.297 | 10.76, 19.36, 31.19 |
| 6.95$r$A | — | 9.033, 17.822, 26.611 | 9.971, 28.339, 32.557 |
| 7.51$r$A | — | 8.301, 16.602, 23.715 | 8.921, 17.843, 26.240 |

**2.2. Autooscillatory Flow upon Interaction of a Jet with an Infinite Target.** An infinite target was located in a steady flow with the following parameters: $M$A $= 2.098$, $n = 4.785$, $\gamma = 1.4$, and $\theta$A $= 4°$. The calculation was performed on a uniform grid $\Delta x = \Delta y = 0.05r$A. The calculation results for $h = 6.95r$A are plotted in Figs. 4 and 5. The nozzle-exit radius $r$A was assumed to be equal to the nozzle-exit radius in the experiment of [10]. Figure 4 shows pressure oscillations on the target at the stagnation point. Figure 5 shows the frequency spectrum of the process $p(t)$.

The numerical frequencies obtained in a series of calculations and the experimental data of [10] are listed in Table 1. We note that the calculated and experimental frequencies are in good agreement (within 5–7%). In the numerical solution (for an infinite target), high-frequency components of the spectrum (28 kHz and higher) were obtained; their amplitude was higher than in the experiments. In our opinion, the reason is the use of the perfect (inviscid) gas model and the insufficient scheme viscosity of the numerical method used, and also the time lag of the measurement equipment in the physical experiment [10].

Note that the numerical and experimental results were also in good agreement in the case of a steady flow formed upon interaction of a supersonic jet with an infinite target and in the case of an autooscillatory flow with known characteristics upon interaction of the jet with a target of the order of the nozzle-exit diameter [10].

## 3. DISCUSSION OF RESULTS

An analysis of the results obtained showed that they are in good agreement with the results of physical experiments [5, 10], for example, in terms of the frequency of pressure oscillations at the stagnation point on the target. The mechanism of self-induced oscillations was studied. It has the following features (noted previously in the literature). A peripheral supersonic flow possessing a high stagnation pressure pinches the central subsonic flow, and a peripheral stagnation point is formed. The closure of the central zone of the flow leads to the accumulation of the gas in this region and the displacement of the central shock toward the nozzle, which involves an increase in pressure upstream of the target with subsequent unclosing of this zone.

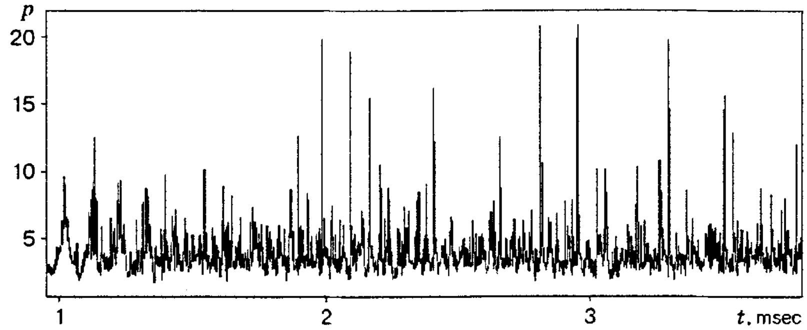


Fig. 4

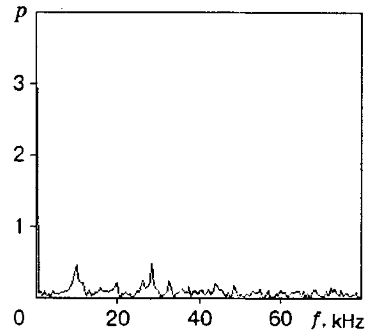


Fig. 5

The low scheme viscosity of the numerical method used does not allow one to damp high-frequency processes in the closed central zone. At the same time, it is insufficient for smoothing significant nonphysical shocks ("noise") on the target at the stagnation point (see Figs. 3 and 5). Nevertheless, the frequency characteristics obtained by means of the Fourier analysis correspond to experimental data.

It should be noted that, as the target size increases, the distance at which the peripheral stagnation point is displaced also increases, and hence, the size of the closed central zone increases too. In this case, high-frequency components appear in the spectrum. Methods with a high scheme viscosity may fail to reproduce these components in numerical simulation (see, for example, [6]).

## CONCLUSIONS

The results obtained confirm the satisfactory accuracy and flexibility of the numerical model used to solve the problem of unsteady interaction of a supersonic axisymmetric jet with a normally positioned planar target of different dimensions. The calculated results correspond to experimental and numerical data on the existence of an autooscillatory regime within a certain range of the parameters of the problem.

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